Quintessence reconstruction of the new agegraphic dark energy model

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In this paper we implement the new agegraphic dark energy model with quintessence field. We demonstrate that the new agegraphic evolution of the universe can be described completely by a single quintessence field. Its potential as a function of the quintessence field is reconstructed numerically. In particular, the analytical solution of the new agegraphic quintessence dark energy model (NAQDE) is approximately obtained in the matter-dominated epoch. Furthermore, we investigate the evolution of the NAQDE model in the $\omega - \omega'$ phase plane. It turns out that by quantum corrections, the trajectory of this model lies outside the thawing and freezing regions at early times. But at late times, it enters the freezing regions and gradually approaches to a static cosmological constant state in the future. Therefore the NAQDE should belong to the freezing model at late times. For comparison, we further extend this model by including the interaction between the NADE and DM and discuss its evolution in the $\omega - \omega'$ phase plane.

I. INTRODUCTION

Recent astronomical observations indicate that the universe is undergoing accelerated expansion at the present time [1]. Nowadays it is the most accepted idea that a mysterious dominant component, dark energy (DE) with negative pressure, leads to this cosmic acceleration. However, the nature and cosmological origin of DE still remain enigmatic at present. It is not clear yet whether DE can be described by a cosmological constant which is independent of time, or by dynamical scalar fields such as quintessence, K-essence, tachyon, phantom, ghost condensate or quintom. At the same time, the so-called fine-tuning problem and coincidence problem still confuse us.

To shed light on these fundamental and difficult problems, some interesting DE models were proposed recently, including holographic dark energy model [2] and agegraphic dark energy model [3, 4]. The former is motivated from the holographic hypothesis. The later is constructed in the light of the Károlyházy relation [5] and a corresponding energy fluctuations of space-time. Therefore, although a complete theory of quantum gravity is not established yet today, we still can make some attempts to investigate the nature of dark energy according to some principles of quantum gravity. The holographic dark energy model and the agegraphic model are just such examples, which are originated from some considerations of the features of the quantum theory of gravity. That is to say, the holographic and agegraphic dark energy model possess some significant features of quantum gravity.

On the other hand, as we all admitted, the scalar field model is an effective description of an underlying theory of dark energy. Scalar fields naturally arise in par-

ticle physics including supersymmetric field theories and string/M theory. Therefore, scalar field is expected to reveal the dynamical mechanism and the nature of dark energy. However, although fundamental theories such as string/M theory do provide a number of possible candidates for scalar fields, they do not uniquely predict its potential $V(\phi)$. Therefore it becomes meaningful to reconstruct $V(\phi)$ from some dark energy models possessing some significant features of the quantum gravity theory, such as holographic and agegraphic dark energy model.

Some works have been investigated on the reconstruction of the potential $V(\phi)$ in holographic dark energy models. For instance we refer to [6, 7]. In this paper, we intend to reconstruct the potential of quintessence $V(\phi)$ for agegraphic dark energy models.

Let us first start with a close look at the agegraphic dark energy model. According to the Károlyházy relation, the distance t in Minkowski spacetime cannot be known to a better accuracy than

$$\delta t = \lambda t_p^{2/3} t^{1/3}, \qquad (1)$$

where λ is a dimensionless constant of order unity. We use the units $\hbar = c = k_B = 1$ throughout this work. Thus $l_p = t_p = 1/M_p$ with l_p , t_p and M_p being the reduced Planck length, time and mass respectively.

Following [8, 9], corresponding to Eq.(1) a length scale t can be known with a maximum precision δt determining thereby a minimal detectable cell $\delta t^3 \sim t_p^2 t$ over a spatial region t^3. Such a cell represents a minimal detectable unit of spacetime over a given length scale t. If the age of the Minkowski spacetime is t, then over a spatial region with linear size t (determining the maximal observable patch) there exists a minimal cell t^3, the energy of which due to time-energy uncertainty relation can not be smaller than

$$E_{\delta t^3} \sim t^{-1}. \qquad (2)$$

Therefore, the energy density of metric fluctuations of

Minkowski spacetime is given by

$$\rho_q \sim \frac{E_{\delta t^3}}{\delta t^3} \sim \frac{1}{t_p^2 t^2} \sim \frac{M_p^2}{t^2}, \qquad (3)$$

which for $t_0 \sim H_0^{-1}$ gives pretty good value for the present dark energy. We refer to the original papers [8, 9] for more details.

Here some arguments are given on cosmological implications of the Károlyházy uncertainty relation (1) and the energy density (3). First, the Károlyházy relation (1) obeys the holographic black hole entropy bound [9]: the relation (1) gives a relation between δl (UV cutoff) and the length scale l (IR cutoff) of a system, $\delta l \sim l_p^{2/3} l^{1/3}$; the system has entropy

$$S \le (\frac{l}{\delta l})^3 \sim (\frac{l}{l_p})^2 \sim S_{BH}, \qquad (4)$$

which is less than the black hole entropy with horizon radius l. Therefore, the Károlyházy uncertainty relation (1) is a reflection of interplay between UV scale and IR scale in effective quantum field theory [10]. The microscopic energy scales of quantum mechanics and the macroscopic properties of our present universe are intimately connected. Second, the energy density (3) is dynamically tied to the large scales of the universe, thus violating naive decoupling between UV scale and IR scale. The appearance of both the Planck length and the largest observable scale in the energy density (3) seems to suggest that the dark energy is due to an entanglement between ultraviolet and infrared physics [11]. Therefore, we expect that the interplay between UV scale and IR scale can give us some clues about what is the reason that quantum gravity effects are still valid today at large distance scales. Some expectations are born out by explicit constructions of effective field theories from string theory [10].

Based on the energy density (3), a so-called agegraphic dark energy (ADE) model was proposed in [3]. There, as the most natural choice, the time scale t in Eq.(3) is chosen to be the age of the universe

$$T = \int_0^a \frac{da}{Ha}, \qquad (5)$$

where a is the scale factor of our universe and $\mathrm{H} \equiv \dot{a}/\mathrm{a}$ is the Hubble parameter, where a dot denotes the derivative with respect to cosmic time.

Later, a new model of ADE was proposed in [4], where the time scale in Eq.(3) is chosen to be the conformal time η instead of the age of the universe. This new agegraphic dark energy (NADE) contains some new features different from the ADE [4, 12, 13] and overcome some unsatisfactory points. For instance, the ADE suffers from the difficulty to describe the matter-dominated epoch while the NADE resolved this issue.

In this paper we intend to present a more detailed investigation on the features of the NADE models through

a quintessence reconstruction. Our paper is organized as follows. We first present brief review on the NADE in section 2. Then we demonstrate a correspondence between the NADE scenario and quintessence dark energy model in section 3. The potential of the new agegraphic quintessence is reconstructed numerically. In addition, we provide an analytical solution of the NAQDE in the matter-dominated epoch. In section 4, we investigate the evolution of the NAQDE model in the $\omega - \omega'$ plane and obtain some important characteristics on the NAQDE model. For comparison we consider the NADE with interaction and investigate its evolution in the $\omega - \omega'$ plane in section 5. Finally, the summary follows in section 6.

II. REVIEW OF THE NEW AGEGRAPHIC DARK ENERGY

In the NADE model [4], the energy density of NADE is

$$\rho_q = \frac{3n^2 M_p^2}{\eta^2}, \qquad (6)$$

where the conformal time η is given by

$$\eta \equiv \int \frac{dt}{a} = \int \frac{da}{a^2 H}. \qquad (7)$$

Note that $\dot{\eta} = 1/a$.

The numerical factor $3n^2$ is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, the effect of curved spacetime (since the energy density in Eq.(3) is derived for Minkowski spacetime), and so on. In [13], H. Wei and R. G. Cai find that the coincidence problem can be solved naturally in the NADE model provided that n is of order unity. In addition, they constrain NADE by using the cosmological obervations of SNIa, CMB and LSS. The joint analysis gives the best-fit parameter (with 1σ uncertainty) $n = 2.716^{+0.111}_{-0.109}$. And in [14], K. Y. Kim et al. argued that the NADE model could describe the matter-dominated (radiation-dominated) universe in the far past only when the parameter n is chosen to be $n > n_c$, where the critical values are determined to be $n_c = 2.6878(2.5137752)$ numerically.

A flat FRW (Friedmann-Robertson-Walker) universe composed of the NADE ρ_q and the pressureless matter ρ_m is governed by the Friedmann equation

$$3M_p^2 H^2 = \rho_m + \rho_q. \qquad (8)$$

Introducing the fractional energy densities $\Omega_i \equiv \rho_i/(3M_p^2 H^2)$ for i = m and q, then one finds

$$\Omega_q = \frac{n^2}{H^2 \eta^2}. \qquad (9)$$

Combining Eqs.(6), (7), (8) and (9), and using the energy conservation equation $\dot{\rho_m} + 3H\rho_m = 0$, we find that
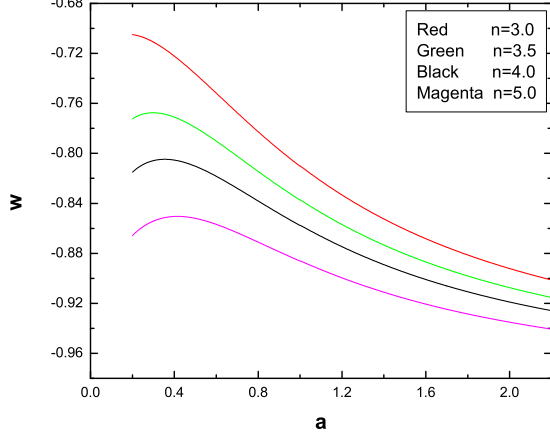
FIG. 1: the evolutions of the equation of state of the NADE. Here we take $\Omega_{q0} = 0.73$, and show the cases $n = 3.0, 3.5, 4.0$ and 5.0. Clearly, the cases always evolve in the region $-1 \leq \omega_q \leq -\frac{2}{3}$, and converge to -1.

the equation of motion for Ω_q is given by

$$\frac{d\Omega_q}{da} = \frac{\Omega_q}{a}(1 - \Omega_q)(3 - \frac{2}{n}\frac{\sqrt{\Omega_q}}{a}). \quad (10)$$

We can also rewrite Eq.(10) as

$$\frac{d\Omega_q}{dz} = -\Omega_q(1 - \Omega_q)[3(1 + z)^{-1} - \frac{2}{n}\sqrt{\Omega_q}], \quad (11)$$

where $z = 1/a - 1$ is the redshif of the universe, and we have set the present scale factor $a_0 = 1$. This equation describes the behavior of the NADE completely.

From the energy conservation equation $\dot{\rho}_q + 3H(\rho_q + p_q) = 0$, as well as Eqs.(6) and (9), it is easy to find that the equation-of-state of NADE $\omega_q \equiv p_q/\rho_q$ is given by

$$\omega_q = -1 + \frac{2}{3n}\frac{\sqrt{\Omega_q}}{a}, \quad (12)$$

or

$$\omega_q = -1 + \frac{2}{3n}\sqrt{\Omega_q}(1 + z). \quad (13)$$

As is referred in [4], $-1 \leq \omega_q \leq -\frac{2}{3}$. Thus, the NADE can be described by the quintessence. As an illustrative example, we plot in Fig.1 the evolutions of the equation of state (11) of the NADE with the cases $n = 3.0, 3.5, 4.0$ and 5.0. It is clear to see that ω_q always evolves in the region $-1 \leq \omega_q \leq -\frac{2}{3}$, and converges to -1.

III. NEW AGEGRAPHIC QUINTESSENCE

In this section, we link the new agegraphic dark energy model with quintessence fields, forming a new agegraphic quintessence dark energy model. Then we numerically simulate the evolution of the quintessence field

with explicit forms of the potential $V(\phi)$. The analytical solution of the NAQDE in the matter-dominated epoch is also obtained.

A. Reconstructing new agegraphic quintessence dark energy

The energy density and pressure for the quintessence field are as following

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (14)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (15)$$

Then one can obtain

$$V(\phi) = \frac{1 - \omega_\phi}{2}\rho_\phi, \quad (16)$$

$$\dot{\phi}^2 = (1 + \omega_\phi)\rho_\phi. \quad (17)$$

Moreover, from the equation of FRW $3M_p^2 H^2 = \rho_m + \rho_\phi$, we can obtain

$$E(a) \equiv \frac{H(a)}{H_0} = (\frac{\Omega_{m0}}{(1 - \Omega_\phi)a^3})^{1/2}, \quad (18)$$

where Ω_{m0} denotes the fractional energy density of pressureless matter today.

Using Eq.(18), one can rewrite Eqs.(16) and (17) respectively

$$\frac{V(\phi)}{\rho_{c0}} = \frac{1}{2}(1 - \omega_\phi)\Omega_\phi E^2, \quad (19)$$

$$\frac{\dot{\phi}^2}{\rho_{c0}} = (1 + \omega_\phi)\Omega_\phi E^2, \quad (20)$$

where $\rho_{c0} = 3M_P^2 H_0^2$ is critical density of the universe at present.

Now we suggest a correspondence between the NADE and the quintessence scalar field, namely, we identify ρ_ϕ with ρ_q. Then, the quintessence field acquires the new agegraphic nature such that E, Ω_ϕ and ω_ϕ are given by Eqs.(11), (13) and (18). Without loss of generality, we assume $\dot{\phi} > 0$ in this paper [6, 15, 16]. Then, the derivative of the scalar field ϕ with respect to the scale factor a can be given by

$$\frac{\frac{d\phi}{da}}{M_p} = \frac{\sqrt{3(1 + \omega_\phi)\Omega_\phi}}{a}. \quad (21)$$

Consequently, we can easily obtain the evolutionary form of the field by integrating the above equation
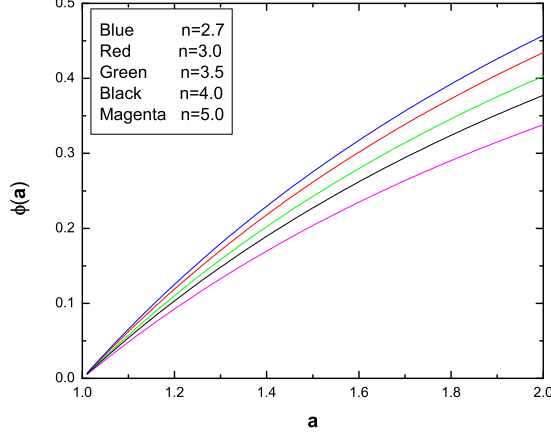
$$\phi(a) = \int_1^a \frac{d\phi}{da}da, \quad (22)$$

FIG. 2: The revolutions of the scalar-field $\phi(a)$ for the NAQDE, where ϕ is in unit of M_p. We take here $\Omega_{mo} = 0.27$.

FIG. 3: The reconstruction of the potential for the NAQDE, where ϕ is in unit of M_p and $V(\phi)$ in ρ_{c0}. We take here $\Omega_{m0} = 0.27$.

where the field amplitude at the present epoch ($a = 1$) is fixed to be zero, namely $\phi(1) = 0$.

In this way, we establish a new agegraphic quintessence dark energy model and reconstruct the potential of the NAQDE.

B. The numerical solution of the NAQDE

The quintessence models with different potential forms have been discussed widely in the literature. For the NAQDE model constructed in this paper, the potential $V(\phi)$ can be determined by Eqs. (19), (20) and (21), and the evolution of Ω_ϕ and ω_ϕ are determined by Eqs. (10) and (12) respectively. The analytical form of the potential $V(\phi)$ is hard to be derived due to the complexity of these equations, but we can obtain the new agegraphic quintessence potential numerically. According to Eqs.(21) and (22), $\phi(a)$ is displayed in Fig.2. The reconstructed quintessence potential $V(\phi)$ is plotted in Fig.3. Selected curves are plotted for the cases of $n = 2.7, 3, 3.5, 4$ and 5, and the present fractional matter density is chosen to be $\Omega_{mo} = 0.27$. From Fig.2 and Fig.3, we can see that the reconstructed quintessence potential is steeper in the early epoch and becomes very flat near today. Consequently, the scalar field ϕ rolls down the potential with the kinetic energy $\dot\phi$ gradually decreasing. Furthermore, we can also find that the n is smaller, the potential $V(\phi)$ is more flat near today. In the next section, we will discuss the approximate solution of the model in the matter-dominated epoch.
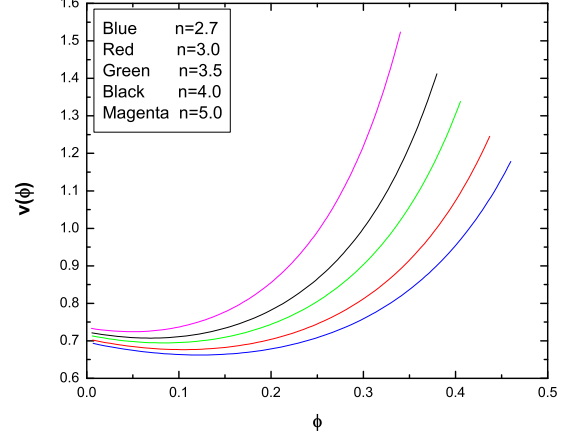
C. Approximate solution of the NAQDE in the matter-dominated epoch

As shown in section 4, the quantum effect is more obvious at early times than at late times. Therefore it is intriguing to considerate the form of the potential $V(\phi)$ in the matter-dominated epoch.

As pointed out in [4], In matter-dominated epoch, $\omega_\phi \approx -2/3$, $\Omega_\phi = \frac{n^2}{4}a^2$. From Eqs.(21) and (19), one can obtain respectively

$$\phi = -\frac{nM_p}{2}a + \alpha, \qquad (23)$$

$$V(a) = \frac{5}{24}\rho_{c0}\Omega_{m0}n^2\frac{a^{-3}}{a^{-2} - \frac{n^2}{4}}, \qquad (24)$$

where α is an integration constant.

Combining Eqs.(23) and (24), the potential $V(\phi)$ can be derived as

$$V(\phi) = \frac{\beta(\alpha - \phi)^{-3}}{M_p^2(\alpha - \phi)^{-2} - 1}, \qquad (25)$$

where $\beta = \frac{5}{48}\rho_{c0}\Omega_{m0}n^3M_p^3$.

It is intriguing to see the expansion of the potential $V(\phi)$ around $\phi = \alpha$. Setting $\varphi = \alpha - \phi$, the potential around $\varphi = 0$ can be expanded as

$$V(\varphi) = \frac{\beta}{M_p^3}[(\frac{\varphi}{M_p})^{-1} + \frac{\varphi}{M_p} + (\frac{\varphi}{M_p})^3 + (\frac{\varphi}{M_p})^5 + (\frac{\varphi}{M_p})^7 + \cdots].$$
$$(26)$$

This leads us to surmise that, in reality, $V(\phi)$ around $\phi = \alpha$ might be considered as an effective potential resulting from the combination of some different fields.

From Eq.(26), we can see that the first term is tracking potential and the other terms are not tracking potential when $\omega_\phi < \omega_m$[17]. This implies that the potential of the NAQDE in the matter-dominated epoch is not tracking potential. In fact, we can easily find that $\Gamma(\phi)$ does not always satisfy $\Gamma(\phi) > 1$, where $\Gamma(\phi) = \frac{V(\phi)''V(\phi)}{(V(\phi)')^2}$. Therefore the potential of the NAQDE in the matter-dominated epoch is not a tracking solution[17].

IV. THE EVOLUTION OF THE NAQDE IN THE $\omega - \omega'$ PLANE

Recently, many authors have investigated the evolution of quintessence dark energy models in the $\omega - \omega'$ plane [18, 19, 20]. According to different regions in the $\omega - \omega'$ plane, the models can be classified into two types which are called thawing and freezing models [17, 18]. Distinguishing the thawing class of dark energy from the freezing class would reveal important clues to the nature of the new physics [18, 19, 20].

In [18], R.R.Caldwell and E.V.Linder have analyzed the following potentials for thawing behavior: $V(\phi) = M^{4-\lambda}\phi^\lambda$, where $\lambda > 0$ and $V(\phi) = M^4 \exp(-\beta\phi/M_p)$. They have also analyzed the following potentials for freezing behavior: $V(\phi) = M^{4+\lambda}\phi^{-\lambda}$ and $V(\phi) = M^{4+\lambda}\phi^{-\lambda}\exp(\alpha\phi^2/M_p^2)$ for $\lambda > 0$. They point out that if the result lies outside the thawing and freezing regions then one may have to look beyond simple explanations, perhaps to even more exotic physics such as a modification of Einstein gravity. In what follows, we shall study the cosmological dynamics of the NAQDE model in the $\omega - \omega'$ plane. For convenience, we will identify ω with ω_q and ω_ϕ.

At first, let us consider the expression of ω', where ω' is the variation with respect to the e-folding time $N \equiv \ln a$. From (12), one can obtain

$$\frac{d\omega}{dz} = \frac{2}{3n}\sqrt{\Omega_\phi} + \frac{1}{3n}\frac{1}{\sqrt{\Omega_\phi}}\frac{d\Omega_\phi}{dz}(1+z). \quad (27)$$

On the other hand,

$$\omega' \equiv \frac{d\omega}{dN} = -(1+z)\frac{d\omega}{dz}. \quad (28)$$

Substituting Eq.(27) into Eq.(28), we have

$$\omega' = -(1+z)[\frac{2}{3n}\sqrt{\Omega_\phi} + \frac{1}{3n}\frac{1}{\sqrt{\Omega_\phi}}\frac{d\Omega_\phi}{dz}(1+z)]. \quad (29)$$

By employing Eqs.(11), (13) and (29), we can plot the trajectory of the NAQDE in $\omega - \omega'$ plane, as is shown in Fig.4, where selected curves are plotted for the cases of $n = 3.0, 3.5, 4.0$ and 5.0 respectively. Here we take $\Omega_{\phi 0} = 0.73$.

The blue line is the coasting line $\omega' = 3(1+\omega)^2$ following from constant field velocity, with ω' greater (smaller) than this for field acceleration (deceleration). As shown in Fig.4, we can see that the trajectory of the NAQDE lies below coasting line at late times and the field accelerates.

The thawing region of phase space is defined by

$$1 + \omega \leq \omega' \leq 3(1+\omega). \quad (30)$$

We note that the lower bound $1 + \omega$ is for $\Omega_{m0} < 0.8$ and $\omega < -0.8$ at present.

Alternatively, the freezing region of phase space is defined by

$$3\omega(1+\omega) \leq \omega' \leq 0.2\omega(1+\omega). \quad (31)$$

The upper bound $0.2\omega(1+\omega)$ is for $\Omega_{\phi 0} > 0.6$ and $\omega < -0.8$ at present [18, 19].

Next, we discuss the physical implication of the trajectory in $\omega - \omega'$ plane.

At early times, as shown in Fig.4, the evolutionary trajectory in the $\omega - \omega'$ plane lies outside the thawing and freezing regions. We may ascribe this to the result of quantum corrections. At late times, the trajectory of evolution enters the so-called freezing region, gradually approaching to a static cosmological constant state in the future. Roughly, the NAQDE should belong to the freezing model at late times. Therefore, it turns out that the quantum effect is more obvious at early times than at late times.

As we have seen, the NAQDE model in the $\omega - \omega'$ plane clearly shows the evolutionary character of the dark energy, and the dynamics of the NAQDE can be explored explicitly by reconstruction.

In addition, these results can be used to discriminate different cosmological models and the future astronomical observations can also discriminate the NAQDE models with different parameters [18, 21].

V. THE NADE WITH INTERACTION

Similar to [4, 21, 22], we further extend the NADE by including the interaction between the NADE and a pressureless cold dark matter, and investigate its evolution in $\omega - \omega'$ plane.

A. The NADE with interaction

We consider the interaction between NADE and DM such that ρ_m and ρ_ϕ respectively satisfy

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (32)$$

$$\dot{\rho}_\phi + 3H(1+\omega_\phi)\rho_\phi = -Q, \quad (33)$$

where Q denotes the interaction term and can be taken as $Q = 3b^2 H\rho$ with b^2 the coupling constant[23, 24]. A more general consideration about the interaction term can be found in[25].
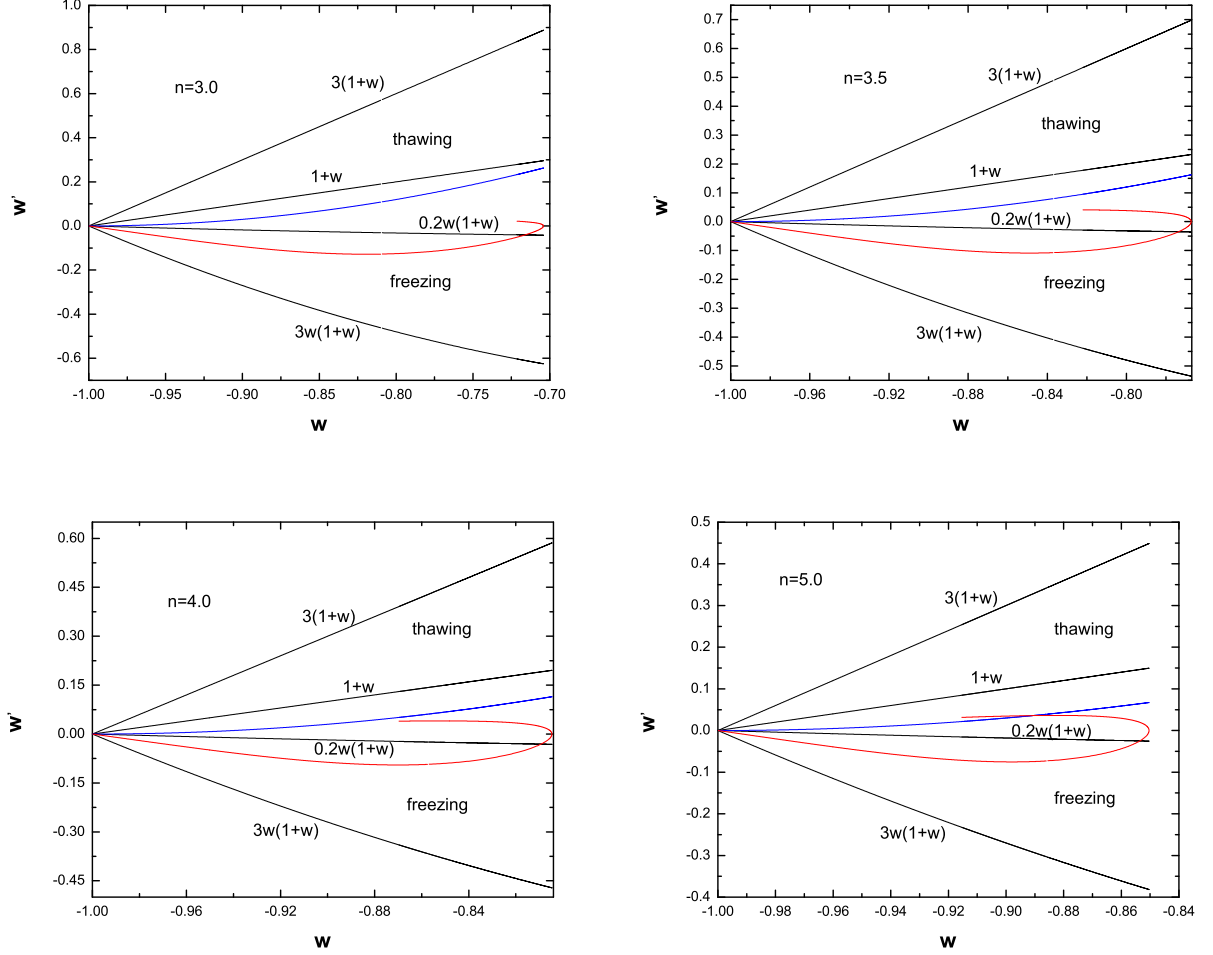
FIG. 4: The trajectory of the NAQDE in $\omega - \omega'$ plane, where selected curves are plotted for the cases of $n = 3.0, 3.5, 4.0$ and 5.0, respectively. Here we take $\Omega_{\phi 0} = 0.73$.

In this case, by using Eqs.(8), (9), (6) and (32), we find that the equation of motion for Ω_ϕ is given by

$$\frac{d\Omega_\phi}{da} = \frac{\Omega_\phi}{a}[(1 - \Omega_\phi)(3 - \frac{2}{n}\frac{\sqrt{\Omega_\phi}}{a}) - 3b^2], \qquad (34)$$

or

$$\frac{d\Omega_\phi}{dz} = -\Omega_\phi\{(1-\Omega_\phi)[3(1+z)^{-1} - \frac{2}{n}\sqrt{\Omega_\phi}] - 3b^2(1+z)^{-1}\}, \qquad (35)$$

From Eqs.(32), (9) and (6), we obtain the EoS as (we can also refer to [4])

$$\omega = -1 + \frac{2}{3n}\sqrt{\Omega_\phi}\frac{1}{a} - \frac{b^2}{\Omega_\phi}, \qquad (36)$$

or

$$\omega = -1 + \frac{2}{3n}\sqrt{\Omega_\phi}(1+z) - \frac{b^2}{\Omega_\phi}. \qquad (37)$$

As is referred in [4], if $b^2 \neq 0$, from Eq.(36) or (37), one can see that ω can be smaller than -1 or larger than -1. As shown in Fig.5 and Fig.6, we can clearly see that ω can cross the phantom divide.

Furthermore, as shown in Fig.5, we see that for a fixed interaction parameter $b^2 = 0.10$ and $n = 2.0, 2.5, 3.0$, ω is larger than -1 and crosses -1 earlier for larger n. But for $n = 4.0$ and 5.0, we can see that ω is smaller than -1 at early times. From Fig.6, for the constant $n = 3.0$, except for $b^2 = 0.10$, ω is smaller than -1 at early times and crosses -1 then. We can also see that ω crosses -1 again and earlier for larger interaction parameter b^2.

B. The evolution of the NADE with interaction in the $\omega - \omega'$ plane

In this subsection we investigate the evolution of the NADE with interaction in the $\omega - \omega'$ plane.

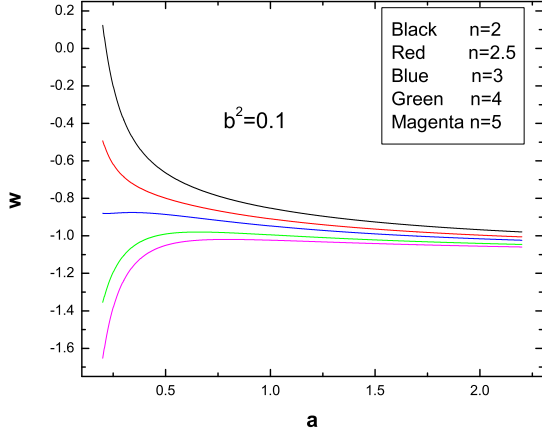At first, let us consider the expression of ω'. From

FIG. 5: Behavior of the equation of state (35). Here we take $\Omega_{q0} = 0.73$, and show the cases for a fixed interaction parameter $b^2 = 0.1$ but for different values of the constant $n = 2.0, 2.5, 3.0, 4.0$ and 5.0.
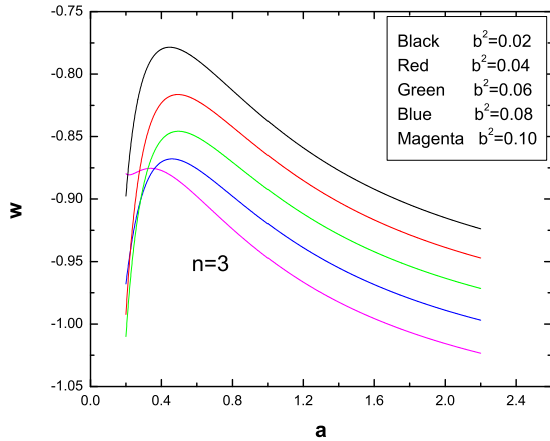

FIG. 6: Behavior of the equation of state (35). Here we take $\Omega_{q0} = 0.73$, and show the cases for a fixed constant $n = 3.0$ but for different values of the interaction parameter $b^2 = 0.02, 0.04, 0.06, 0.08$ and 0.10.

Eq.(37), one obtains

$$\omega' = -(1+z)[\frac{2}{3n}\sqrt{\Omega_\phi} + \frac{1}{3n}\frac{1}{\sqrt{\Omega_\phi}}\frac{d\Omega_\phi}{dz}(1+z) + \frac{b^2}{\Omega_\phi^2}\frac{d\Omega_\phi}{dz}].$$
(38)

By using Eqs.(35), (37) and (38), we can plot in Fig.7 the trajectory of the NADE with interaction in $\omega - \omega'$ plane, where selected curves are plotted for the cases of $n = 3$ and $b^2 = 0.10, 0.08, 0.06, 0.04, 0.02$ respectively.

Here we take $\Omega_{\phi 0} = 0.73$.

As shown in Fig.7, such coupling shift the trajectory of the NAQDE in the $\omega - \omega'$ phase space, allowing for dynamics outside the thawing and freezing regions. We can also see that except for $b^2 = 0.10$, the trajectory lies above the coasting line at early times and lies below the coasting line at late times. It elucidates that the field acceleration at early times and deceleration in its motion at late times. In the future, the trajectory crosses the phantom divide and enters the phantom regions.

VI. SUMMARY

It is fair to claim that the NADE provides a reliable framework for investigating the problem of dark energy, owing to possessing some features of the quantum gravity theory. In this paper, we suggest a correspondence between the new agegraphic dark energy scenario and the quintessence scalar-field model. We have demonstrated that the new agegraphic evolution of the universe can be described completely by a single quintessence field. Its potential is reconstructed numerically. In addition, we have also obtained the approximate solution of the NAQDE in the matter-dominated epoch and analyzed its character. Furthermore, we have investigated the evolution of the NAQDE in the $\omega - \omega'$ phase plane. We find that the quantum effect is more obvious at early times than at late times. For comparison, the evolution of the NADE model with interaction has also been investigated in the $\omega - \omega'$ plane.

This work is a first step towards studying the form of scalar potential and the evolution in the $\omega - \omega'$ plane by quantum corrections. It will also be interesting to further investigate the analytical form of the NAQDE potential by some approximations. In future works, we will extend such investigations including ADE, NADE and holographic dark energy. In all this paper we assume our universe is spatially flat but it is completely possible to show that the parallel analysis could be extended to the spatially closed and hyperbolic universe. We also expect that the further investigation will provide us a more exact picture of dark energy.
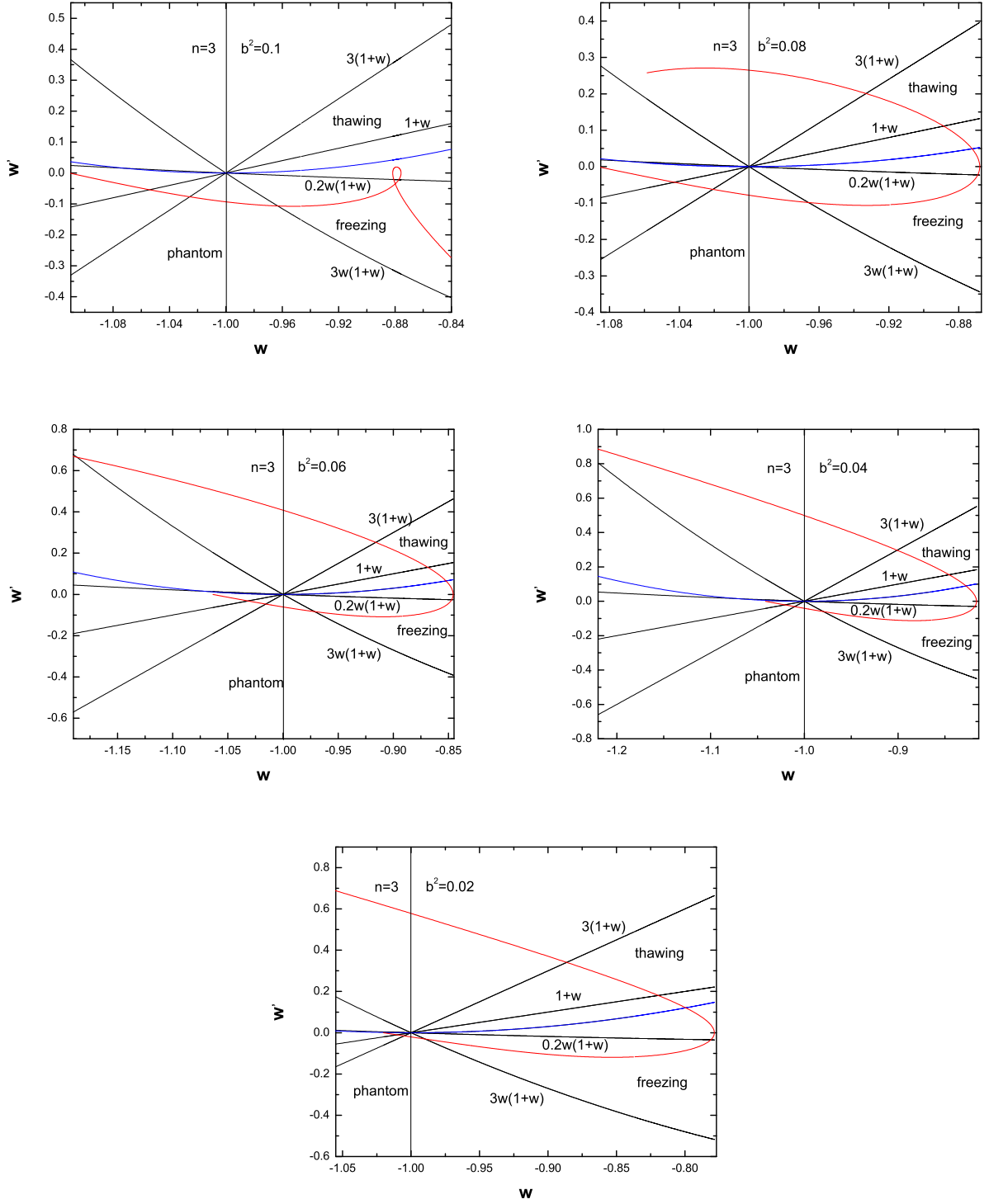
FIG. 7: The trajectory of the NADE with interaction in $\omega - \omega'$ plane, where selected curves are plotted for the cases of a fixed constant $n = 3.0$ but different values of the interaction parameter $b^2 = 0.10, 0.08, 0.06, 0.04$ and 0.02. Here we take $\Omega_{\phi 0} = 0.73$.

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